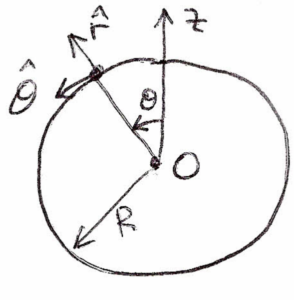
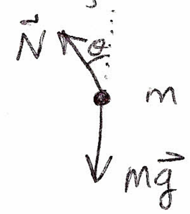
Phys 410
Spring 2013, Prof. Anlage
8 February, 2013

Problem 1. Consider a small frictionless puck perched on the top of a fixed sphere of radius R. If the puck is given a tiny nudge so that it begins to slide down, through what vertical height will it descend before it leaves the surface of the sphere?

Hint: Use conservation of energy to find the puck's speed as a function of height, then use Newton's second law to find the normal force of the sphere on the puck. At what value of this normal force does the puck leave the sphere?



Free body diagram for the puck:



Energy: Kinetic $T = \frac{1}{2}mv^2$

Potential $U = mgz$ (using $U=0$ at the height of the center of the sphere)

$= mgR\cos\theta$

Conservation of Energy

$$\left.\begin{aligned} E_{initial} &= E_{later} \\ E_{initial} &= 0 + mgR \\ E_{later} &= \tfrac{1}{2}mv^2 + mgR\cos\theta \end{aligned}\right\} \quad \text{Hence } T = E - U$$

$$= mgR - mgR\cos\theta$$

$$\frac{mv^2}{2} = mgR(1-\cos\theta) \quad [1]$$

From the free body diagram, and the observation that the puck has centripetal acceleration while on the sphere, Newton's 2nd law says: $m\vec{a} = \vec{F}_{net}$ $\vec{a} = -\frac{v^2}{R}\hat{r}$; $\vec{F}_{net} = \vec{N} + m\vec{g}$

Look at the radial component of the equation:

$$-\frac{mv^2}{R}\hat{r} = -mg\cos\theta\,\hat{r} + N\hat{r}$$

or $\frac{mv^2}{R} = mg\cos\theta - N$. Substitute [1] for mv^2 and solve for N

$N = mg(3\cos\theta - 2)$. The normal force goes to zero (i.e. the puck leaves the sphere) when $\theta = \cos^{-1}(2/3) = 48.2^\circ$

$$\vec{\nabla}\times\vec{F} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ F_x & F_y & F_z \end{vmatrix} = \hat{x}\left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right) - \hat{y}\left(\frac{\partial F_z}{\partial x} - \frac{\partial F_x}{\partial z}\right) + \hat{z}\left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)$$

Problem 2

Is the following force conservative?

$F_x = ayz + bx + c$ $\quad \frac{\partial F_x}{\partial z} = ay \quad \frac{\partial F_x}{\partial y} = az$

$F_y = axz + bz$ $\quad \frac{\partial F_y}{\partial z} = ax + b \quad \frac{\partial F_y}{\partial x} = az$

$F_z = axy + by$ $\quad \frac{\partial F_z}{\partial y} = ax + b \quad \frac{\partial F_z}{\partial x} = ay$

where a, b, c are constants.

$$\vec{\nabla}\times\vec{F} = \hat{x}(ax + b - ax - b) - \hat{y}(ay - ay) + \hat{z}(az - az)$$

$$= \hat{x}(0) - \hat{y}(0) + \hat{z}(0)$$

$$= 0 \quad \checkmark$$

This means that the work calculated from this force is path independent

In addition, $\vec{F} = \vec{F}(x,y,z) = \vec{F}(\vec{r})$ and does not depend on velocity, time, etc.

Hence the two conditions are satisfied $\Rightarrow$ $\vec{F}$ is conservative

Is the following force conservative?

$F_x = -ze^{-x}$ $\quad \frac{\partial F_x}{\partial z} = -e^{-x} \quad \frac{\partial F_x}{\partial y} = 0$

$F_y = \ln z$ $\quad \frac{\partial F_y}{\partial z} = \frac{1}{z} \quad \frac{\partial F_y}{\partial x} = 0$

$F_z = e^{-x} + y/z$ $\quad \frac{\partial F_z}{\partial y} = \frac{1}{z} \quad \frac{\partial F_z}{\partial x} = -e^{-x}$

Hence

$$\vec{\nabla}\times\vec{F} = \hat{x}\left(\frac{1}{z} - \frac{1}{z}\right) - \hat{y}(-e^{-x} + e^{-x}) + \hat{z}(0 - 0)$$

$$= \hat{x}(0) - \hat{y}(0) + \hat{z}(0)$$

$$= 0$$

In addition $\vec{F} = \vec{F}(x,y,z)$ only

Hence $\vec{F}$ is a conservative force